Pre-Thanksgiving...

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Each student is given a worksheet; the recorder collects the worksheets and hands them in at the end of the period. To create the worksheets, I divide an 8.5 x 11 sheet of 1/4" graph paper into 12 equal parts, letting 1 unit = 5 feet. Each part contains a 7 x 13 unit grid (representing the 35 ft x 65 ft backyard). I give each student one 2-sided worksheet.

Some students will mistakenly assume that on the grid, 1 unit = 1 foot. I ask these students to estimate the ratio of the area of the garden to the area of the yard to determine if their layouts are reasonable. The ratio is 240/2275 which is approximately 240/2400 or 1/10. Revisions usually follow.

Students may also need to be reminded that determining the number of posts is not a matter of simply dividing the perimeter by 8. The team needs to discuss this carefully. Hint: ask them if there can be an odd number of posts. (A few days after Thanksgiving can be given to developing an algorithm for determining the number of posts using divisibility and the greatest integer function.)

To finish the project before Thanksgiving, Part I of the project should be completed on the second day. Lend assistance as needed if teams are not drawing sound conclusions after a reasonable time. Students can go on to Parts II and III on the second day, if there's time.

For Part II, the cost of implementing a layout includes the fixed cost of the fertilizer ($175) as well as the variable cost of the fencing and posts, which are determined by the perimeter and dimensions. Students should figure out that although a 12 x 20 layout may be distinguishable from a 20 x 12 layout in the yard, the cost to implement each is the same. In fact there are actually only 7 layouts with different costs. Students may guess or reason correctly that the layouts with the greatest and least perimeters have the greatest and least cost, respectively. Using a team strategy can reduce the amount of work needed in this step.

On the third day, students should complete parts II and III. If a team finishes early, give the recorder an additional problem:

Part IV. Suppose the four requirements listed in Part I are no longer necessary (for example, the dimensions need not be whole numbers and the shape need not be a rectangle), except that the area of the garden must still be (approximately) 240 square feet (to use only one bag of fertilizer). Sketch additional possible layouts. Creativity is encouraged! Label the dimensions and determine the costs. Are any of these less costly than the least-cost layout in Part II? Should cost be the only factor considered?

With the restrictions removed, a few students may realize that a circular garden with a radius approximately the square root of $(240/\pi)$ only requires 7 posts. Others may see that of all rectangular layouts, a square (or nearly square) garden will have the least cost for a given area.

During the project, students may ask whether there is a need for some type of a garden gate. I suggest that a gate could be figured into the cost and the fencing perimeter and number of posts could be recalculated. Most students respond with the recommendation that, for the time being, Miles can easily hurdle the fence.

Finding all the Factors

I first review vocabulary: factor, divisor, multiple, prime and composite numbers, prime factorization. Then we work on the following problems as a class in preparation for the project.

List and count the factors of:

(1) **54** [Ans: 1, 2, 3, 6, 9, 18, 27, 54; 8 factors].
(2) **156** [Ans: 1, 2, 3, 4, 6, 12, 13, 26, 39, 52, 78, 156; 12 factors].

Now try:

(3) **16** [1, 2, 4, 8, 16], **32** [1, 2, 4, 8, 16, 32]

Observe: $16 = 2^4$ and has $5 = 4+1$ factors

(4) Generalize to $\mathbf{2^n}$ [Ans: $1, 2^1, 2^2, \ldots, 2^n$; $n+1$ factors]

Now observe: $54 = 2^1 \times 3^3$,

So the possible factors are:

$2^0 \times 3^0 = 1$ $\quad 2^1 \times 3^0 = 2$
$2^0 \times 3^1 = 3$ $\quad 2^1 \times 3^1 = 6$
$2^0 \times 3^2 = 9$ $\quad 2^1 \times 3^2 = 18$
$2^0 \times 3^3 = 27$ $\quad 2^1 \times 3^3 = 54$

and so 54 has $8 = (1+1)(3+1)$ factors.

Now try:

(5) $156 = 2^2 \times 3^1 \times 13^1$.
[Ans: $(2+1)(1+1)(1+1) = 3 \times 2 \times 2 = 12$]

Generalize to:

(6) $N = 2^a \times 3^b \times 5^c \times 7^d$
[Ans: $(a+1)(b+1)(c+1)(d+1)$]